HIGHER ORDER INTEGRATION ALGORITHM FOR THE BUTTERWORTH FILTER DIFFERENCIAL EQUATIONS

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The present paper presents a higher order integration algorithm for the Butterworth filter differential equations. The classic solution derived from Z transform by using bilinear transformation (trapezoidal operator) gives a second order multi-step algorithm as well known [1]. So, it is necessary to take into account special operators to treat the initial conditions in order to attend the mathematical consistence. The proposed fifth order single step hermitian finite difference operator permits to formulate a higher order single step integration algorithm for the Butterworth filter differential equations [2], for which the initial conditions can be directly treated.

Initially considering the function, its first and second derivative values, the single step fifth orders hermitian finite difference operator is developed. This operator is expressed in terms of a classical notation as [2]:

$$12y_i - 12y_{i+1} + 6\Delta t\dot{y}_i + 6\Delta t\dot{y}_{i+1} + \Delta t^2\ddot{y}_i - \Delta t^2\ddot{y}_{i+1} = 0 + O\left(\Delta t^5\right)$$

where Δt is the time increment. The order of error is obtained taking into account the correspondent Taylor expansions. Following, a fifth order forward lagrangian finite difference operator is derived in order to evaluate higher order derivatives of the signal to be processed to attend mathematical error consistence. That operator is given by [3]:

$$\Delta t.\dot{x}_i = -\frac{137}{60}x_i + 5x_{i+1} - 5x_{i+2} + \frac{10}{3}x_{i+3} - \frac{5}{4}x_{i+4} + \frac{1}{5}x_{i+5}$$

where x represents the signal to be processed.

Application examples show that the proposed algorithm presents results much more accurate than the classic second order solution given in the literature as expected. Table 1 compares some results of a typical example.

T (sec.)	Trapezoidal	Hermitian	Exact
50	0.936973	0.939090	0.939094
52	0.810137	0.807853	0.807852
54	0.400296	0.394410	0.394404
56	-0.149379	-0.156811	-0.156820
58	-0.646872	-0.653254	-0.653263
60	-0.918394	-0.921497	-0.9921502

Table 1 – Numerical Results

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